TM5 7.2 TAKING DERIVATIVES

THE LAGRANGIAN IS

$$L = \frac{1}{2}m\left\{\left[v_0 + a_0 t + l\dot{\theta}\cos\theta\right]^2 + l^2\dot{\theta}^2\sin^2\theta + 2gl\cos\theta\right\}$$

TAKE DERIVATIVES (θ IS THE ONLY GENERALIZED COORDINATE)

$$\frac{\partial L}{\partial \theta} = m\left[(v_0 + a_0 t + l\dot{\theta}\cos\theta)(-l\dot{\theta}\sin\theta) + l^2\dot{\theta}^2\sin\theta\cos\theta + 2gl\sin\theta\right]$$

$$= ml\left[-(v_0 + a_0 t)\dot{\theta}\sin\theta - l\dot{\theta}^2\sin\theta\cos\theta + l\dot{\theta}^2\sin\theta\cos\theta + 2g\sin\theta\right]$$

$$\underline{\frac{\partial L}{\partial \theta} = ml\left[-(v_0 + a_0 t)\dot{\theta} + 2g\right]\sin\theta}$$

$$\frac{\partial L}{\partial \dot{\theta}} = m\left[(v_0 + a_0 t + l\dot{\theta}\cos\theta)(l\cos\theta) + l^2\dot{\theta}\sin^2\theta + 0\right]$$

$$= ml\left[(v_0 + a_0 t + l\dot{\theta}\cos\theta)\cos\theta + l\dot{\theta}\sin^2\theta\right]$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = ml\left\{\left[0 + a_0 + (l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta)\right]\cos\theta + \right.$$

$$\left. + \left[v_0 + a_0 t + l\dot{\theta}\cos\theta\right](-\dot{\theta}\sin\theta) + l\ddot{\theta}\sin^2\theta + 2l\dot{\theta}^2\sin\theta\cos\theta\right\}$$

$$= ml\left[a_0\cos\theta + l\ddot{\theta}\cos^2\theta - l\dot{\theta}^2\sin\theta\cos\theta - v_0\dot{\theta}\sin\theta - a_0 t\dot{\theta}\sin\theta + \right.$$

$$\left. - l^2\dot{\theta}^2\cos\theta\sin\theta + l\ddot{\theta}\sin^2\theta + 2l\dot{\theta}^2\sin\theta\cos\theta\right]$$

$$= ml\left[a_0\cos\theta + l\ddot{\theta}(\underbrace{\cos^2\theta + \sin^2\theta}_{=1}) - 2l\dot{\theta}^2\sin\theta\cos\theta + \right.$$

$$\left. - v_0\dot{\theta}\sin\theta - a_0 t\dot{\theta}\sin\theta + 2l\dot{\theta}^2\sin\theta\cos\theta\right]$$

$$\underline{\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = ml\left[a_0\cos\theta + l\ddot{\theta} - v_0\dot{\theta}\sin\theta - a_0 t\dot{\theta}\sin\theta\right]}$$

LAGRANGE'S EQUATION THEN GIVES

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = 0$$

$$ml\left\{(-v_0 - a_0 t)\dot{\theta}\sin\theta + 2g\sin\theta - a_0\cos\theta - l\ddot{\theta} + v_0\dot{\theta}\sin\theta + \right.$$

$$\left. + a_0 t\dot{\theta}\sin\theta\right\} = 0$$

$$\Rightarrow \quad 2g\sin\theta - a_0\cos\theta - l\ddot{\theta} = 0$$